

Pedagogical Implications of Bhaskaracharya's Lilavati

Sudhakar C Agarkar

VPM's Academy of International Education and Research.

Dr Bedekar Vidya Mandir, Naupada

Thane, 400602, Maharashtra State, India.

Abstract

Bhaskaracharya, a celebrated Indian mathematician wrote a mathematical treatise entitled "Lilavati" in 12th century. It contains theorems and problems in elementary Arithmetic, Algebra and Geometry. Written in poetic form with ample use of alliterations metaphors, pun and tenderness the treatise displays various pedagogical characteristics like gender equity, motivational anecdotes, rapport building, content relevance etc. As a part of a year long celebration of 900th birth anniversary, 65 workshops were conducted based on Lilavati in the academic year 2014-15 both in rural as well as urban parts of India. The participants for the workshops were drawn from primary, secondary as well as tertiary level of education. It has been observed that the teaching techniques advocated in Lilavati proved useful in creating interest among students towards mathematics. They not only remove the fear of mathematics but motivates them to undertake problem solving exercises with zeal and enthusiasm. This paper brings out pedagogical characteristics of Bhaskaracharya's Lilavati and indicates their relevance for teaching school mathematics in 21st century.

Introduction

Bhaskaracharya, a great Indian mathematician was born in 1114 at Bijjal Bid close to western ghat in India. Bhaskaracharya, meaning Bhaskar the teacher (the Sanskrit word acharya means a teacher) displays many useful pedagogical qualities through his writings. Bhaskaracharya was a prolific writer and wrote profusely in mathematics and astronomy. Lilavati, a mathematical treatise written sometimes in the middle of 12th century can be used as an example. The entire book is written in a poetic form with profuse use of

alankaras (figures of speech) like pun, alliteration, metaphors and similes in his writing. There are about 270 verses in the book. The book begins with an invocation addressed to Lord Ganesha seeking his blessings. I would like to quote it in Sanskrit with English translation by H. T. Colebrook.

प्रीतिभक्तजनस्य यो जनयते विघ्नं विनिघ्नन स्मृतः ।

तं वृन्दारकवृद्धं दितपदं नत्वा मतंगाननम् ॥

पाटीसद्गणितस्य वच्मि चतुरप्रीतिप्रदां प्रस्फुटाम् ।

संक्षिप्ताक्षरकोमलामलपदैर्लालित्यलीलवतीम् ॥

Having bowed to the deity, whose head is like an elephant; whose feet are adorned by gods; who, when called to mind, relieves his votaries from embarrassment; and bestows happiness on his worshippers; I propound this easy process of computation, delightful by its elegance, perspicuous with words concise, soft and correct, and pleasing to the learned.

Lilavati was used as a text book of mathematics for many years. It has been translated into many languages. The book, though old, stands out as an important mathematical treatise for school education. The pedagogical guidelines suggested in the book are relevant even today after 900 years. This paper presents the pedagogic analysis of Lilavati and discusses its relevance for the teaching of mathematics in the present century.

Pedagogical Characteristics

Lilavati, called patiganit, mainly discusses concepts in arithmetic, elementary Algebra and Geometry. A critical look at this treatise brings out many pedagogical characteristics that are useful and relevant in teaching these concepts. Some of them are presented below.

Gender Equality

It is said that Bhaskaracharya named the book after his daughter Lilavati. Whether this legend is true or not it is evident that he advocated gender equality through his writings. At many places in Lilavati he refers to readers as bale, sakhe, charming girl, etc. An example in this context is given below.

बाले बालकुरंगलोलनयने लीलावति प्रोच्यताम ।
 पञ्च्येकमिता दिवाकरगुणा अंकः कति स्युर्यदि ॥
 रौँपस्थानविभागखंडगुणने कल्पासि कल्याणिनि ॥
 छिन्नस्तेन गुणेन ते च गुणिता अंकाः कति स्युर्वद ॥

A beautiful and dear Lilavati, whose eyes are like fawn's, tell me the numbers resulting from one hundred thirty five taken into twelve, if thou be skilled in multiplication by whole or parts whether by subdivision of form or separation of digits.

One must keep in mind that 12th century Indian society discouraged girls to take formal schooling. Even when the education was opened to female members they opted for softer subjects and avoided hard disciplines like mathematics. It was certainly revolutionary on the part of Bhaskaracharya to address mathematics problems to a female member through his writings in those days.

Motivation

Deliberate attempt to motivate learners to undertake problem solving is the second pedagogic characteristics that is evident from Lilavati. Bhaskaracharya tries to create interest among the readers through a variety of means. In some cases he presents the problem through a story. In other cases he creates interesting situations. A problem involving snake and peacock can be considered as an example in this category.

अस्ति स्तंभतले बिलं तदुपरि क्रीडाशिखंडी स्थितः।
 स्तम्भेहस्तनवोच्छ्रिते त्रिगुणितस्तंभप्रमाणांतरे ॥
 दृष्ट्वाहिं बिलमात्रजंतमपतत तिर्यक स तस्योपरि ।
 क्षिप्रं ब्रूहितयोर्बिलात्कतिमितैः साम्येन गत्योर्युतिः ॥

A snake hole is at the foot of a pillar, nine cubits high, and a peacock is perched on its summit. Seeing the snake at a distance of thrice the pillar gliding towards the hole, he pounces obliquely upon him. Say quickly how many cubits from the snake's hole they meet, both proceedings an equal distance.

Motivation is considered to be an essential component of learning. It is more so in mathematics,

which is quite abstract in nature. Unless the person is motivated mathematical concepts would not be fixed even if the teacher has completed teaching. Bhaskaracharya seems to be aware of this pedagogical fact nine centuries ago. Hence, he gives adequate importance to motivate the learner even before a task is given through a variety of means.

Rapport Building

The entire book Lilavati is written in first person active voice. The author addresses the reader with loving words like mitra, sakhi, bale, etc. At many places he addresses the readers as mathematician and instigates them to undertake problem solving. Here is an example from Lilavati.

एकविंशतियुतं शतद्वम | यद्गुणं गणकं पंचषष्टियुक्तं ॥

पंचवर्जितशतद्वयोद्धृतम | शुद्धिदमेति गुणकं वदशु तत ॥

“Say quickly, mathematician, what is that multiplier, by which two hundred and twenty-one being multiplied, and sixty-five added to the product, the sum divided by a hundred and ninety-five becomes exhausted.”

Most of the times the book on mathematics prescribed for formal study is written in a dull and dry style. Bhaskaracharya avoids this style and attempts to make the book interesting to read. Open endedness is the hallmark of Lilavati. Bhaskaracharya has solved hardly any problem in his book. He suggests multiple ways of dealing with a problem and allows the reader to choose the method of his or her choice. At some places he provides necessary hints but does not force the reader to follow a specific method.

Content Relevance

Problems posed in Lilavati are related to daily life activities like income, area, stacking of bricks, etc. The problems are composed taking animate as well as inanimate objects found around. Thus snakes, bees, swans, elephants, etc. are referred to in the description. Here is one such example from Lilavati.

यातं हंसकुलस्य मूलदशकं मेघागमे मानसं 1

प्रोड्डीय स्थलपद्मिनीवनमगदष्टांशको भस्तटात 11

बालेबाल्मृणालशालिनि जले केलिक्रियालालसम 1

दृष्टं हंसयुगत्रयं च सकलां यूथस्य संख्या वद ॥

Out of the swans in a certain lake, ten times the square root of their number went away to Manasa Sarovara when rains started, and one eighth the number went away to the forest Sthala Padmini. Three pairs of swans remained in the tank, engaged in water sports. What is the total number of swans?

Studies show that students dislike mathematics as they find it irrelevant to their daily lives. Realising this fact Bhaskaracharya has made deliberate efforts to pose problems that are relevant to the lives of readers. At the same time Bhaskaracharya has arranged the sequence in such a way that students develop all the necessary prerequisites knowledge and skills before reaching a major problem in the section. This helps them build confidence and to deal with the main problems without any difficulty.

Field Experiences

In order to celebrate the 900th birth anniversary of Bhaskaracharya the Vidya Prasarak Mandal (VPM), Thane has decided to organise an international conference on the life and work of Bhaskaracharya in September 2014. In addition, workshops were planned for students and teachers based on Lilavati. In a span of one year 65 workshops were conducted in different schools and colleges in the states of Maharashtra, Madhya Pradesh and Andhra Pradesh in India. These workshops are conducted in two parts. The first part was devoted to familiarising the participants with the rich tradition of Mathematics in India. The second part of the workshop is used to deal with selected problems from Lilavati. These problems are chosen taking into account the educational background of the participants (Ma, 1999).

The experiences of conducting these workshops have been quite positive. Students, at different levels, find the task of solving problems from Lilavati very interesting. It is notable that the participants engage themselves in the task for almost three hours. On many occasions they come forward to show their method of problem solving on the blackboard. Due to educational practice that is followed in the school they do not appreciate the open ended approach advocated by Bhaskaracharya to begin with (Shimada2007). Soon however, they start realising the importance of this method and come out with novel method of dealing with a given problem. An experience in this context is worth sharing.

While solving the snake and peacock problem mentioned above I could notice innovative approaches used by students in a rural school of India. Standard method that is followed in dealing with the problem is to apply Pythagoras theorem and find out the value of requisite distance. One boy however, came out 12 as the answer quickly. On enquiry he revealed that he tried to look for a triad having number 9 in it. Another

boy went on adding square number to 81 (square of 9) and checked whether the answer is a square number. Through this method he also came out with an answer 12 quickly.

Conclusion and implications

It is clear that the pedagogical guidelines advocated by Bhaskaracharya in Lilavati are relevant even today. Efforts must be made to practice them in day to day teaching. It must be noted that the methods adopted by Bhaskaracharya would work both for first generation learners as well as traditional learners (Agarkar, 1997). They would enable us to make people mathematically literate and develop problem solving among them. Constructivistic approach, Situational learning and experiential learning are the buzz words of present day pedagogy. Without saying it explicitly, Bhaskaracharya had adopted all of these pedagogic aspects in his writings. It is high time that we make use of our traditional knowledge to improve teaching of mathematics in the world.

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